

A Method of Finding the Maxima and Minima of a Function of a Single Discrete Variable

by

Nirmalya Basu

(Email ID: nirmalya1basu@gmail.com)

Let it be required to find the maxima and the minima of a function $f(n)$ where n is a discrete variable. To do so, we take the function $f(x)$ where x is a continuous real variable. We note that $f(x) = f(n)$ for all $x = n$. We also assume that $f(x)$ and $f'(x)$ are both continuous and differentiable at all points x in their domains of definition. Then, we can use the methods of calculus of continuous real variables for finding the maxima and minima of $f(x)$. Let a maximum has occurred at $x = x_1$. If x_1 is equal to some value of n , say n_1 , then $f(n_1)$ is the maximum of $f(n)$. If, on the other hand, x_1 is equal to some value other than those of n , we should evaluate $f(x)$ at $x = n_2$ and $x = n_3$, where n_2 is just less than x_1 and n_3 is just greater than x_1 , i.e., we should evaluate $f(n_2)$ and $f(n_3)$. Then, whichever of $f(n_2)$ and $f(n_3)$ is greater is the maximum of $f(n)$. Now, let us consider a minimum. Let a minimum of $f(x)$ has occurred at $x = x_1'$. If x_1' is equal to some value of n , say n_1' , then $f(n_1')$ is the minimum of $f(n)$. If, on the other hand, x_1' is equal to some value other than those of n , we should evaluate $f(x)$ at $x = n_2'$ and $x = n_3'$, where n_2' is just less than x_1' and n_3' is just greater than x_1' , i.e., we should evaluate $f(n_2')$ and $f(n_3')$. Then, whichever of $f(n_2')$ and $f(n_3')$ is lesser is the minimum of $f(n)$. This method can be used to find all the maxima and minima of $f(n)$.

If $f(n)$ be given as a table of values, then we can find the continuous function $f(x)$ (where x is a continuous real variable) such that $f(x) = f(n)$ for all $x = n$ by interpolation. After that, we can use the above method for finding the maxima and the minima of $f(n)$.